Kinetic theory of nonequilibrium Luttinger liquids

Dmitry Polyakov

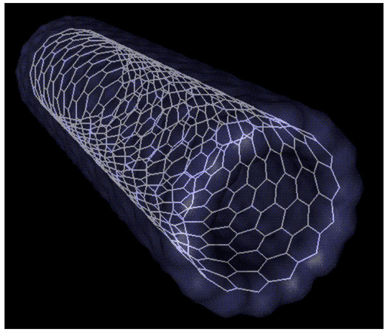
Institut für Nanotechnologie, Forschungszentrum Karlsruhe GmbH, Germany

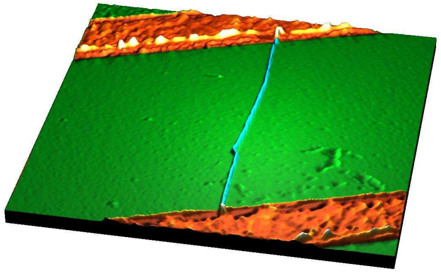
Dmitry Bagrets *(Forschungszentrum Karlsruhe)*
Igor Gornyi *(Forschungszentrum Karlsruhe & Ioffe Institute)*

Outline:

- ▷ **Nonequilibrium : Why 1D electrons are special**
- ▷ **Inhomogeneous Luttinger liquid out of equilibrium :**
  Kinetic theory approach
- ▷ **Equilibration in a Luttinger liquid**

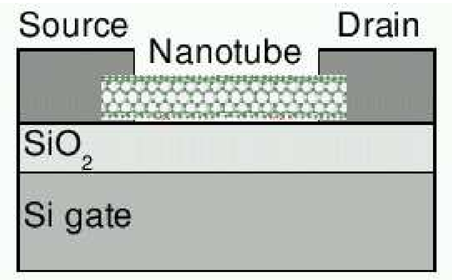
Single-channel quantum wires (*aka* Nanowires)



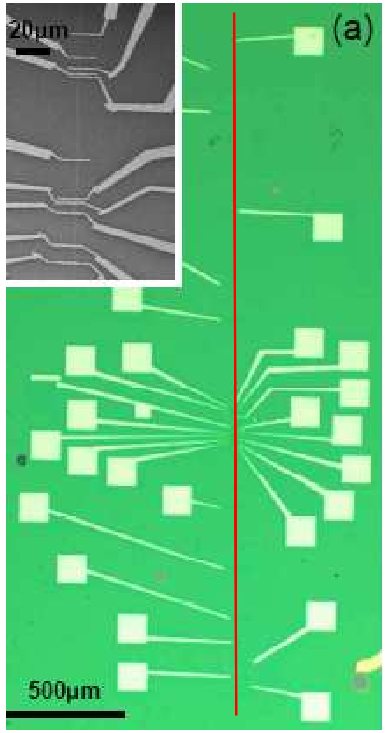


- Carbon nanotubes
- Semiconductor quantum wires
- Quantum Hall edges
- Polymer nanofibers
- Metallic nanowires
- . . .

Single-wall carbon nanotube = cylindrical roll of graphene



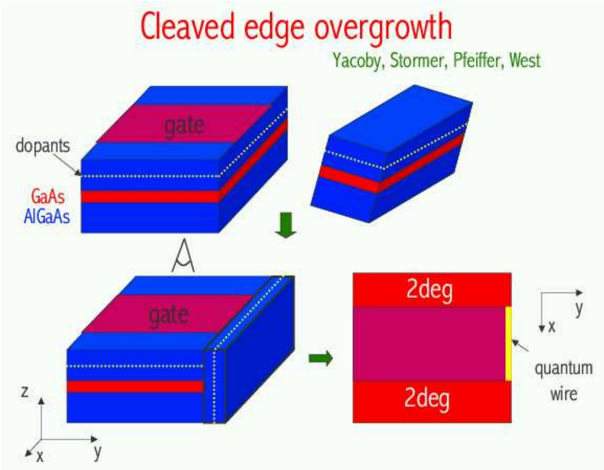
$$\boldsymbol{R \sim \underline{1\,\mathrm{nm}}\ ,\ \ L \sim 1\,\mu\mathrm{m} - \underline{1\,\mathrm{mm}}}$$



$$\textbf{Metallic nanotubes}:\ \textbf{Mean free path}\ \ \boldsymbol{l \sim \underline{1\,\mu\mathrm{m}}}$$

From **Purewal et al., PRL '07**

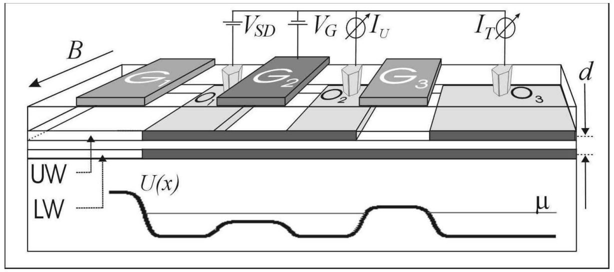
Single-channel quantum wires (*aka* Nanowires)



Atomic-precision "cleaved-edge" single-channel GaAs wires at the intersection of two quantum wells

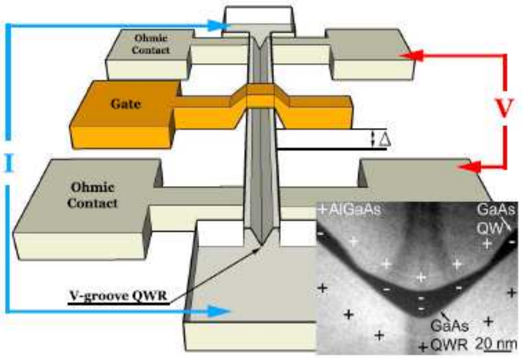
$R \sim \underline{10\,\text{nm}}$

- **Carbon nanotubes**
- **Semiconductor quantum wires**
- **Quantum Hall edges**
- **Polymer nanofibers**
- **Metallic nanowires**
- **...**



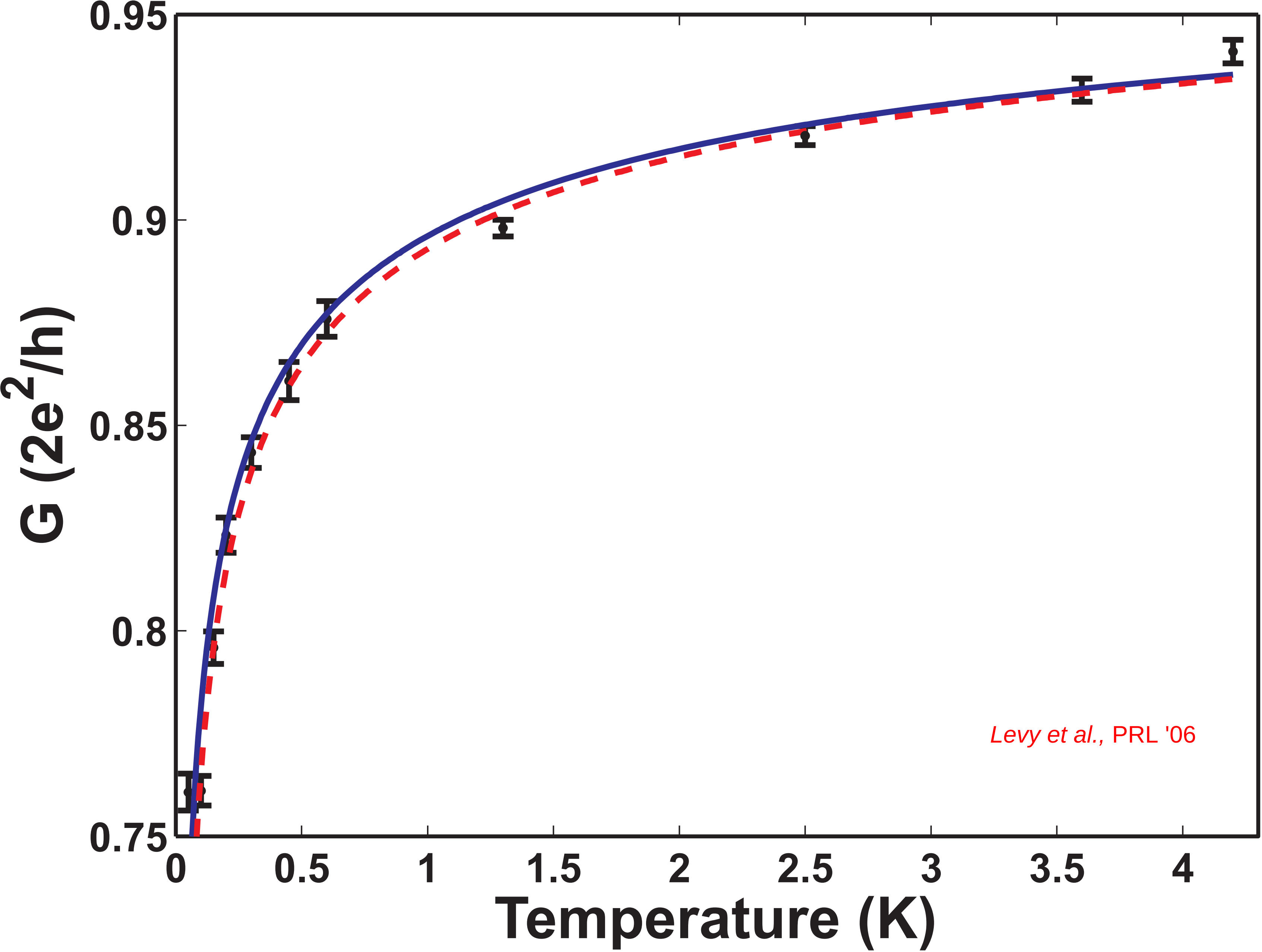
From **Auslaender et al.,** *Spin-charge separation and localization in one dimension,* **Science '05**

V-groove nanowire



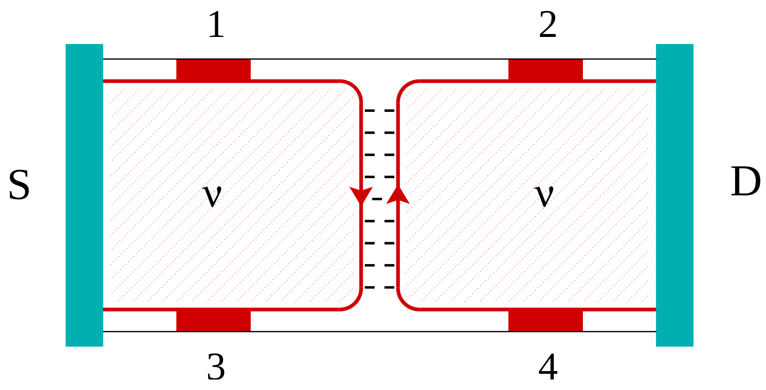
From **Levy et al.,** *Luttinger-liquid behavior in weakly disordered quantum wires,* **PRL '06**

Semiconductor nanowires : Mean free path $l \sim \underline{10\,\mu\text{m}}$



0.95
0.9
0.85
0.8
0.75
G (2e^2/h)
0
0.5
1
1.5
2
2.5
3
3.5
4
Temperature (K)
Levy et al., PRL '06

Single-channel quantum wires (*aka* Nanowires)



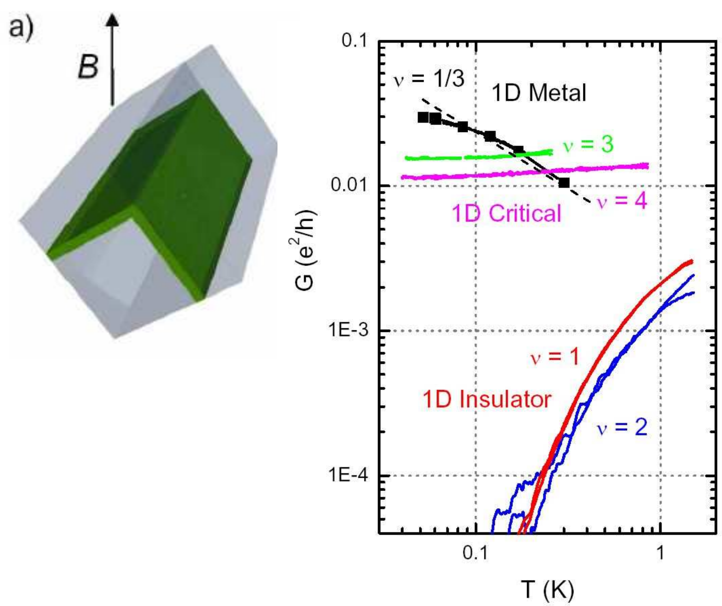
Quantum-Hall line junctions: longest ($L \sim \underline{1\,\text{cm}}$) single-channel GaAs quantum wires

- Carbon nanotubes
- Semiconductor quantum wires
- Quantum Hall edges
- Polymer nanofibers
- Metallic nanowires
- ...

backscattering disorder = random interedge tunneling

1D barrier in 2D: Kang et al., Nature '00; Yang et al., PRL '04

L-shaped quantum wells: Grayson et al., APL '05, PRB '07



Mean free path in 1D controlled continuously by magnetic field

From Grayson et al., PRB '07

Interacting electrons in 1D are special !

- ***Fermi liquid*** $\longrightarrow$ ***Luttinger (“non-Fermi”) liquid***

  exact excitations : plasmons (+spinons)
  “proper language” = bosons

  Electron operator: $\psi_{\pm} \longrightarrow e^{i(\theta \pm \phi)}$ with $\partial_x \theta = \pi \Pi$, $[\phi(x), \Pi(x')] = i\delta(x - x')$

  right/left movers

  Fermionic (quartic) : $H_F = v_F \sum_{\mu=\pm} i\mu \int dx\, \psi_\mu^\dagger \partial_x \psi_\mu \;+\; H_{\mathrm{ee}}$

  Bosonic (quadratic) : $H_B = \frac{c}{2} \int dx \left[(\partial_x \phi)^2 + \frac{v_F^2}{u^2} (\partial_x \theta)^2 \right]$

  elastic 1D string (u — plasmon velocity)

- ***Clean Luttinger liquid : completely integrable***

Integrability prohibits relaxation to equilibrium

Interacting electrons in 1D are special !

Landau Fermi-liquid theory : equilibration through the creation of electron-hole pairs

Landau-quasiparticle (lifetime)$^{-1}$:

$$\tau_{\mathrm{ee}}^{-1} \sim \alpha^2 \frac{T^2}{\epsilon_F} \qquad \text{(3D)}$$

$$\tau_{\mathrm{ee}}^{-1} \sim \alpha^2 \frac{T^2}{\epsilon_F} \ln \frac{\epsilon_F}{T} \qquad \text{(2D)}$$

α —interaction constant

energy transfer (energy of the e-h pairs) in 3D or 2D : $\omega \sim T$

1D :
"Golden Rule"

$$\tau_{\mathrm{ee}}^{-1} \sim \alpha^2 v_F T \int d\omega \int dq \, \delta(\omega - v_F q)\delta(\omega + v_F q) \sim \alpha^2 T$$

RL scattering (R^2 and L^2 scattering cancelled via exchange)

$\omega = 0$!

Equilibration in a Luttinger liquid is only possible in the presence of inhomogeneities

Energy relaxation in single-channel quantum wires

(biased by a finite source-drain voltage, etc.)

- ***Clean Luttinger liquid: if excited, never decays to equilibrium (characterized by temperature)***

- ***Inhomogeneous Luttinger liquid:***

  ▷ **backscattering disorder (impurities)** $\longleftarrow$ **this talk**

  Bagrets, Gornyi & Polyakov '08

  $$U_b(x)\psi_+^\dagger(x)\psi_-(x) \;\leftrightarrow\; U_b(x)e^{2i\phi(x)}$$ **(Luttinger → Sine-Gordon)**

  ▷ **inhomogeneous (forward scattering) interaction strength**

  (cf. scattering on the contacts to the leads) *Gutman, Gefen & Mirlin '09*

- ***Beyond the Luttinger liquid model:***

  finite curvature of the dispersion relation $\longrightarrow$ ***triple collisions***

  Lunde, Flensberg & Glazman '07

  curvature-induced relaxation weak in $T/\epsilon_F \ll 1$

Localization and dephasing in a disordered Luttinger liquid

- ***No e-e interaction*** $(\alpha = 0)$ **:** ***disordered 1D gas = Anderson insulator***

  loc. length $\sim$ mean free path

  $\sigma(T) \equiv 0$ for $\forall\, T$ —at vanishing coupling to the external world (phonons, etc.)

- ***T–dependent screening of disorder:*** ***free path*** $l(T) = u\tau(T) \propto T^{2\alpha}$

  Physics: scattering off Friedel oscillations (but beyond Hartree-Fock)

- ***Dephasing of the localization in a Luttinger liquid***

  Gornyi, Mirlin & Polyakov '05

  Physics: scattering on disorder-damped plasmons ("Münchhausen effect")

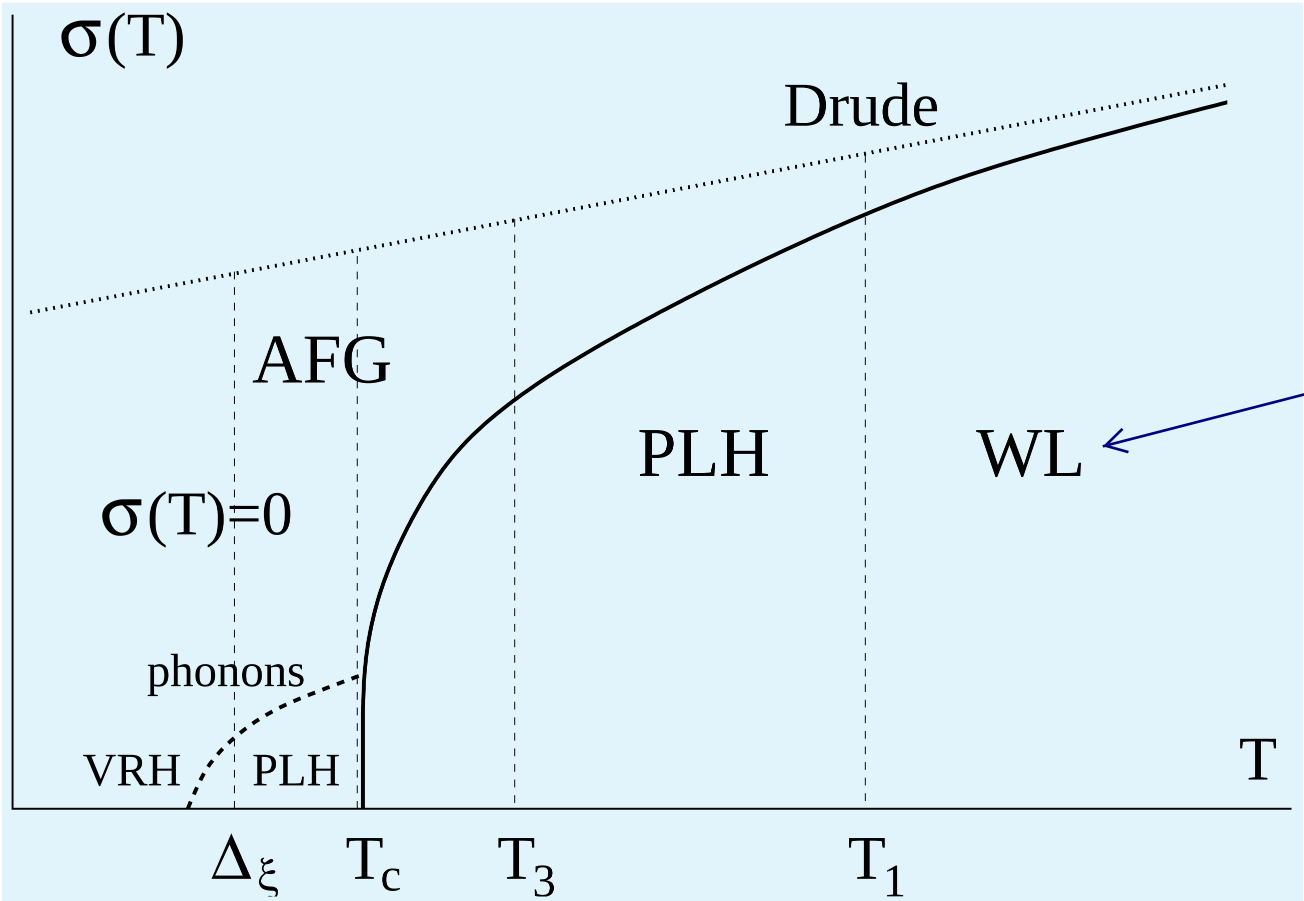
  ▷ weak-localization dephasing rate : $\tau_\phi^{-1} = \alpha(\pi T/\tau)^{1/2}$

  ▷ out-scattering (lifetime)$^{-1}$ $\tau_{\rm ee}^{-1} = \pi\alpha^2 T \gg \tau_\phi^{-1}$

  (but does not manifest itself in the conductivity)

  ▷ $\tau_\phi^{-1} \gg \tau^{-1} \quad \left(T \gg T_1 = \frac{1}{\alpha^2\tau}\right)$ $\longrightarrow$ localization correction is small:

  $$\frac{\Delta\sigma}{\sigma} = -\frac{1}{4}\left(\frac{\tau_\phi}{\tau}\right)^2 \ln\frac{\tau}{\tau_\phi} \propto \frac{1}{\alpha^2 T}\ln(\alpha^2 T)$$



this talk is here (metal)

log-log scale

$\boldsymbol{T \gg T_1}$**: metal**

$\boldsymbol{T_c < T \lesssim T_1}$**: onset of the localization**

$\boldsymbol{T < T_c}$**: insulator** *Gornyi, Mirlin & Polyakov '05*

Basko, Aleiner & Altshuler '06

Q: How to describe a Luttinger liquid out of equilibrium?

• ***Bosons only: Nonequilibrium bosonization?***

Fendley, Ludwig & *Saleur '95*: **integrable single-impurity problem**
conductance through a single impurity between equilibrium leads biased by a voltage

Furusaki & *Nagaosa '93*, *Egger* & *Grabert '96*: **"partial nonequilibrium"**
equilibrium leads with a finite bias: formally exact, but the complexity rapidly becomes untreatable with increasing number of impurities

Gutman, Gefen & *Mirlin '09*: **nonequilibrium without impurities**

...

• ***Kinetic theory that deals with kinetic equations for distribution functions?***
Kinetic-theory approach: one has to introduce the distribution functions of both fermions and bosons. $\longrightarrow$ **This work.**

Real-time fermionic action

$$S\{\psi\} = S_0 + S_{\rm ee} + S_b$$

linear dispersion: $S_0 = i\int_K dt \int dx \sum_{\mu=\pm} \overline{\psi}_\mu \left(\partial_t - \mu v_F \partial_x\right)\psi_\mu$

Keldysh contour

e-e interaction: $S_{\rm ee} = \frac{1}{2}\sum_{\mu\mu'}\int_K dt \int dx_1\, dx_2\, n_\mu(x_1) V(x_1 - x_2) n_{\mu'}(x_2)$

weak forward e-e scattering: $\alpha = \frac{1}{\pi v_F}\int dx\, V(x) \ll 1$

disorder: $S_{\rm b} = \int_K dt \int dx \left[U_b(x)\overline{\psi}_-\psi_+ + \text{h.c.}\right]$

$$\left\langle U_b^*(x) U_b(0)\right\rangle = \frac{v_F}{2\tau}\delta(x)$$

weak backward disorder-induced scattering: $\frac{1}{\epsilon_F \tau} \ll 1$

Nonequilibrium functional bosonization

- ▷ **Hubbard-Stratonovich decoupling for e-e interaction**
  $\longrightarrow$ preserves both fermionic and bosonic degrees of freedom
- ▷ **"quasiclassical" Keldysh Green's function** $\hat{g}(x,t_1,t_2)$
- ▷ **Eilenberger-type equation in the fluctuating decoupling field** $\hat{\varphi}(x,t)$

$$iv_F\partial_x\hat{g} + \left[\left(i\partial_t - \hat{\varphi}\right)\sigma_z - \tfrac{1}{2}\left(U_b\sigma_+ + U_b^*\sigma_-\right)\ ,\ \hat{g}\right] = 0$$

exact in the Luttinger-liquid model

constraint $\hat{g}\circ\hat{g} = \delta(t_1 - t_2)$

- ▷ **averaging over disorder (Born)**

$$iv_F\partial_x\hat{g}_\mu + \left[\left(i\partial_t - \hat{\varphi}\right)\sigma_z - \tfrac{i}{4\tau}\,\hat{g}_{-\mu}\ ,\ \hat{g}_\mu\right] = 0$$

- ▷ **effective action** (cf. *Muzykantskii & Khmelnitskii '95; Kamenev & Andreev '99*)

$$S\{\hat{g},\hat{\varphi}\} = \tfrac{1}{2}\,\mathrm{Tr}\int dx\left[\tfrac{1}{v_F}\left(-i\partial_t + \hat{\varphi}\right)\sigma_z\hat{g} - \hat{g}_0\mathcal{T}^{-1}i\partial_x\mathcal{T} - \tfrac{i}{16v_F\tau}\sigma_+\hat{g}\sigma_-\hat{g}\right]$$

unitary $\mathcal{T}$: φ-induced fluctuations of $g = \mathcal{T}g_0\mathcal{T}^{-1}$ around the noninteracting g_0

S generates the equation of motion for g as its saddle point

Kinetic equation for electrons

$$\left[\partial_t - \alpha(\partial_t \rho_{-\mu})\partial_\epsilon + \mu v_F \partial_x\right] f^\mu_\epsilon = -\frac{1}{2\tau(\epsilon)}\left(f^\mu_\epsilon - f^{-\mu}_\epsilon\right) + \mathrm{St}^\mu_{\mathrm{e-b}}(\epsilon)$$

inelastic electron scattering on the plasmon bath

$$\mathrm{St}^\mu_{\mathrm{e-b}} = \sum_\nu \int d\omega\, I^{\mu\nu}(\omega)\left[f^\nu_{\epsilon+\omega}\left(1-f^\mu_\epsilon\right) - f^\mu_\epsilon\left(1-f^\nu_{\epsilon-\omega}\right)\right]$$

rate of emission of energy (per unit interval of ω), with scattering $\mu \to \nu$

$$I^{\mu\nu}(\omega) = \frac{i}{\pi}\int \frac{dq}{2\pi} V^{\mu\nu}_>(\omega, q)\ \mathrm{Re}D^{\mu\nu}(\omega, q)$$

dynamically screened effective e-e interaction

electron-hole propagator

Poles of the plasmon propagator: $q \simeq \pm\frac{\omega}{u}\left(1 \pm \frac{i}{2\omega\tau}\right)$

close !

Poles of the particle-hole propagator: $q \simeq \pm\frac{\omega}{v_F}\left(1 \pm \frac{i}{2\omega\tau}\right)$

for $\omega\tau \gg 1$

Disorder-induced resonant enhancement of inelastic scattering

Poles of the plasmon propagator: $q \simeq \pm\frac{\omega}{u}\left(1 \pm \frac{i}{2\omega\tau}\right)$

Poles of the particle-hole propagator: $q \simeq \pm\frac{\omega}{v_F}\left(1 \pm \frac{i}{2\omega\tau}\right)$

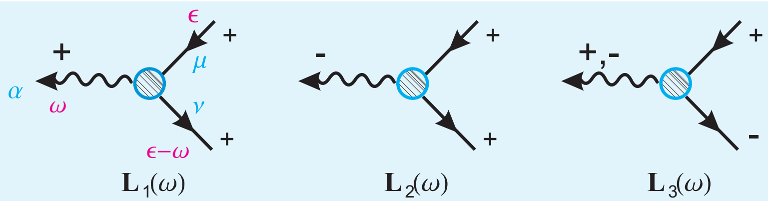
For $\omega \gg T_1 = \frac{1}{\alpha^2\tau}$ **, the poles are well separated (splitting** $\gg$ **broadening):**

$$I^{\mu\nu} = I^{\mu\nu}_{\mathrm{p}} + I^{\mu\nu}_{\mathrm{e-h}}$$

collective poles **electron-hole poles**

$$I^{\mu\nu}_{\mathrm{p}}(\omega) = \sum_\lambda \omega L^{\mu\nu}_\lambda [1 + n^\lambda(\omega)]$$

$$I^{\mu\nu}_{\mathrm{e-h}}(\omega) = \delta_{\mu\nu} \int d\epsilon' L_g(\omega) f^{\mu}_{\epsilon'-\omega}\left(1 - f^{\mu}_{\epsilon'}\right)$$



$$L_2 \propto \alpha^2 \,, \quad L_3 \propto \alpha$$

inelastic scattering kernel

$$L_1 \simeq L_g \simeq \frac{1}{\omega^2\tau}$$

does not depend on the interaction $\alpha \ll 1$**!**

$$\alpha^2/(v_F - u) \sim 1$$

renormalized by g_4

Kinetic equation for plasmons

$$(\partial_t + \mu u \partial_x) n^\mu_\omega = -\tfrac{1-\alpha}{\tau} n^\mu_\omega + \mathrm{St}^\mu_\mathrm{b}(\omega)$$

$$\mathrm{St}^\mu_\mathrm{b}(\omega) = \tfrac{1}{2\tau}\left[\left(1+\tfrac{u}{v_F}\right) N^{\mu\mu}_\omega + \left(1-\tfrac{u}{v_F}\right) N^{-\mu,-\mu}_\omega - 2\alpha N^{\mu,-\mu}_\omega\right]$$

$$N^{\mu\nu}_\omega = \tfrac{1}{2\omega}\int d\epsilon\ \left[f^\mu_\epsilon\left(1 - f^\nu_{\epsilon-\omega}\right) + (\mu \leftrightarrow \nu)\right]$$

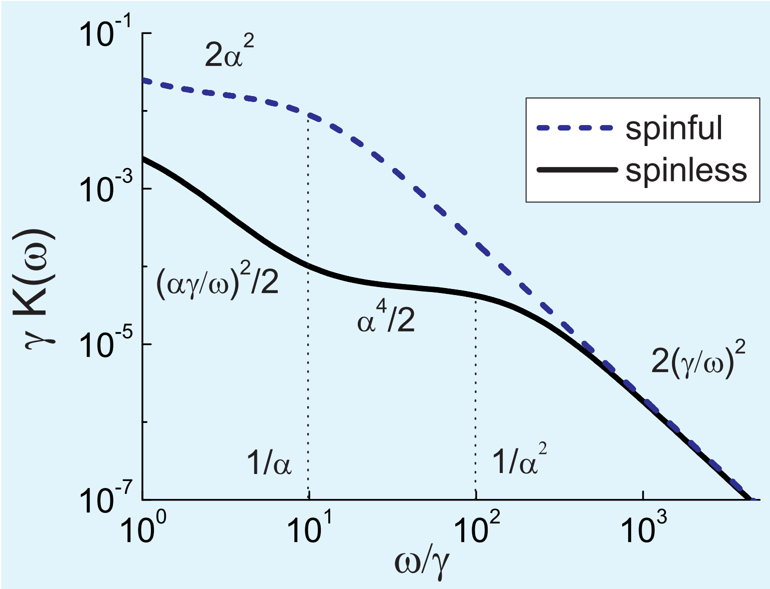
nonequilibrium $N^{\mu\mu}_\omega \neq n^\mu(\omega)$:

boson scattering $\equiv$ creation/annihilation of electron-hole pairs

(anharmonic decay of plasmons and their scattering on each other—weaker)

Equilibration rate in a Luttinger liquid

Total collision kernel $K(\omega)$:



$\omega \gg 1/\alpha^2\tau$

$$K(\omega) \simeq 2/\omega^2\tau$$

$T \gg 1/\alpha^2\tau$

Equilibration rate $$\frac{1}{\tau_E(T)} \sim \frac{1}{T}\int_0^T d\omega\, \omega^2 K(\omega) \sim \frac{1}{\tau}$$

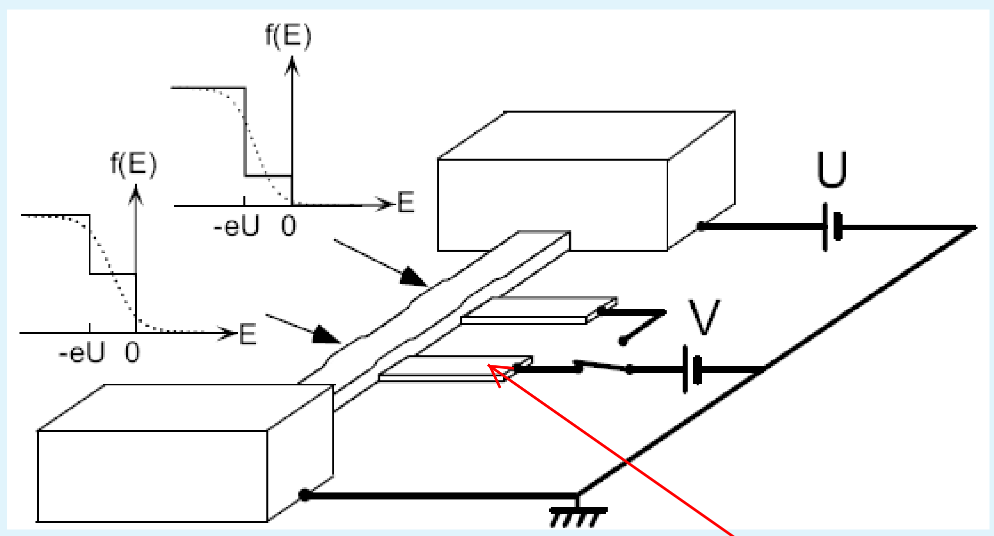
= impurity scattering rate !

temperature and interaction independent !

Out-scattering rate $\tau_{\rm ee}^{-1} \sim T\int_0^T d\omega\, K(\omega) \sim \alpha^2 T$ **—same as at** $\tau^{-1} = 0$

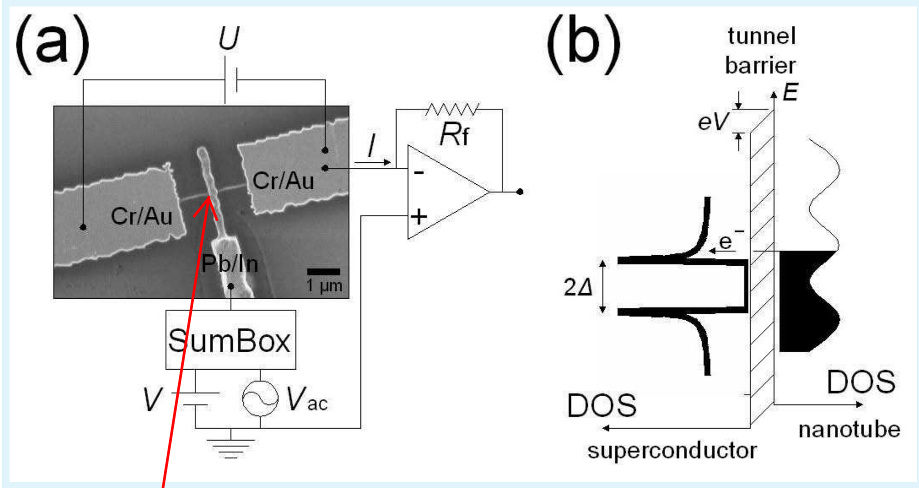
Measuring the nonequilibrium distribution function : Tunneling spectroscopy

Double-step electron distribution function



Pothier et al., PRL'97; Pierre et al., PRL'01

QUASI-1D (MULTICHANNEL) GOLD WIRE



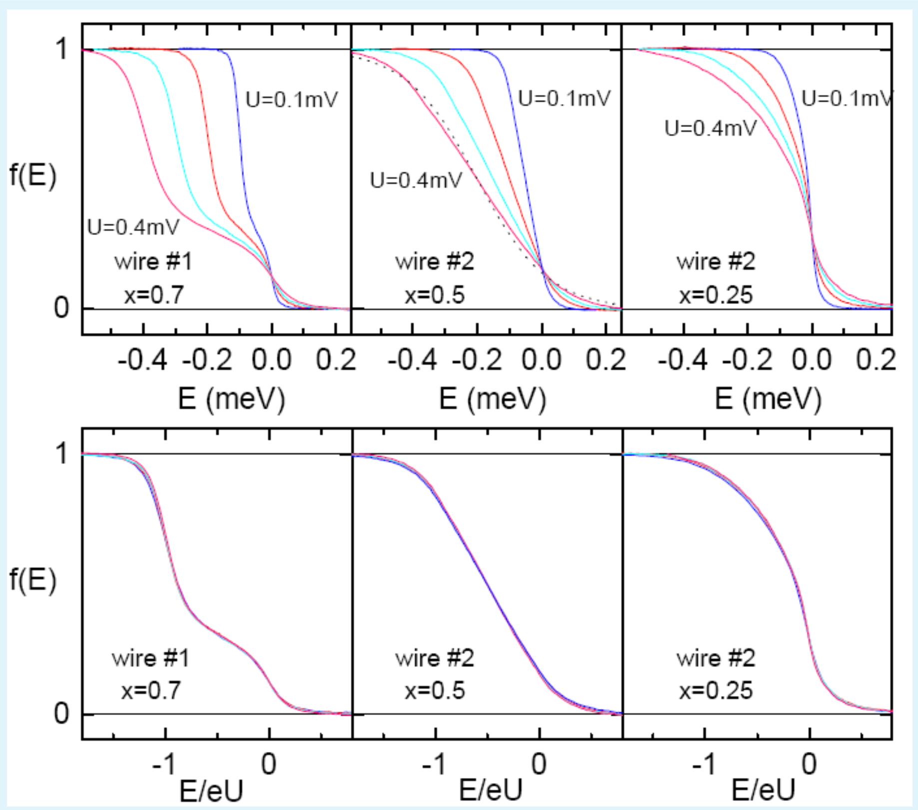
Chen et al., PRL'09

CARBON NANOTUBE WIRE

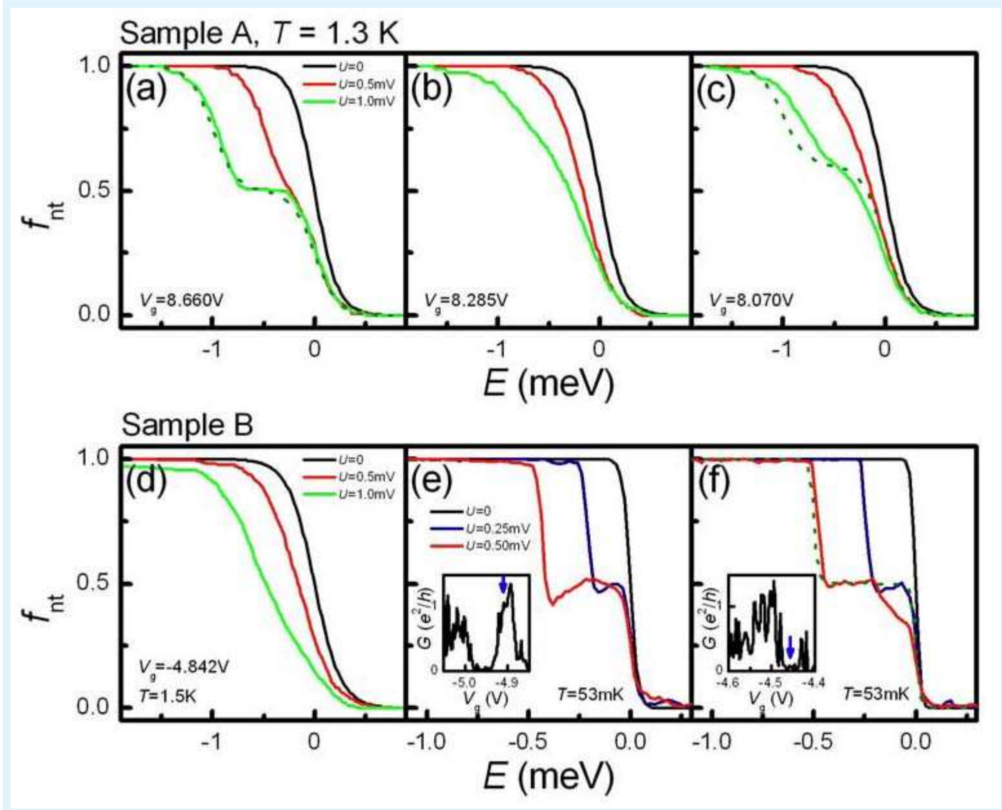
superconducting tips

Measuring the nonequilibrium distribution function : Tunneling spectroscopy

Au wire | **C nanotube**



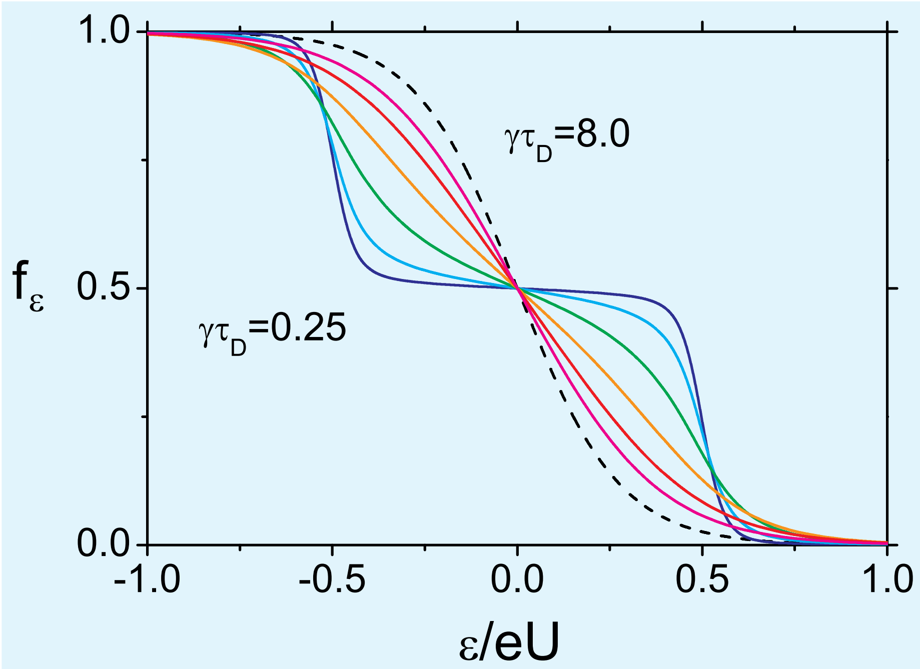
Pierre et al., PRL'01



Chen et al., PRL'09

Electron distribution function in a biased quantum wire: Our theory

f_ϵ in the middle of the wire



$$eU = 40T = 20T_1$$

γ—elastic scattering rate , $\tau_D = L/v_F$—dwell time

- curves are α independent
- characteristic equilibration rate is γ

Summary

KINETIC THEORY APPROACH
TO NONEQUILIBRIUM LUTTINGER LIQUIDS

- ***Formulated kinetic-equation description of a Luttinger liquid out of equilibrium***
- ***Disorder-induced resonant enhancement of inelastic scattering***
- ***Equilibration rate = elastic scattering rate***